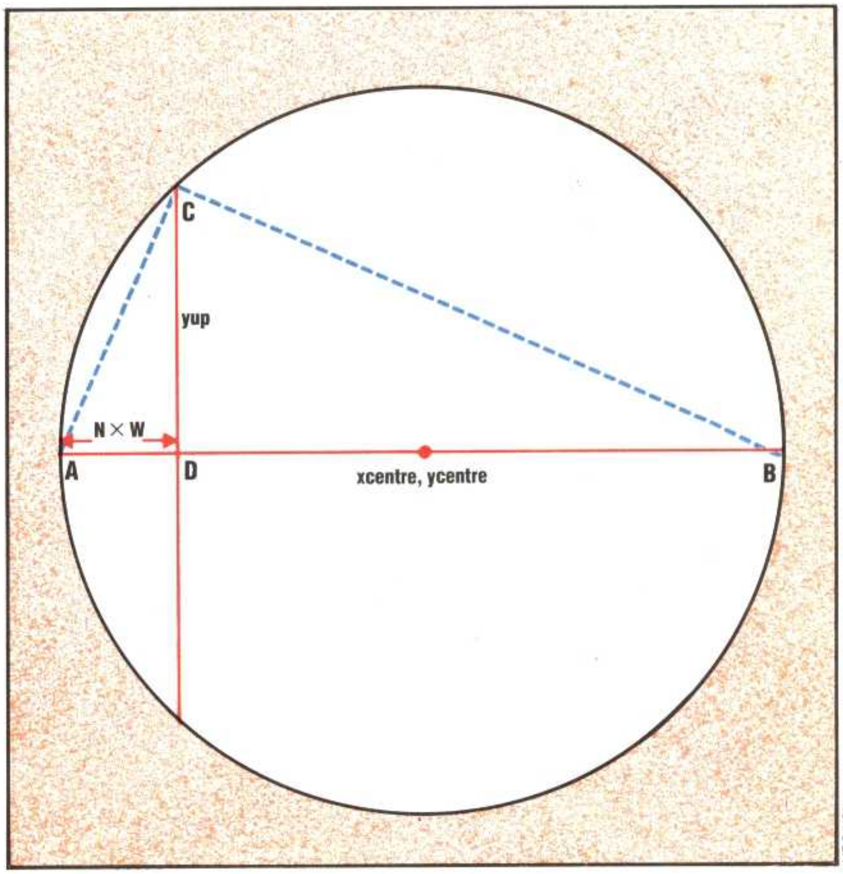
THE RING CYCLE

So far in the course we have built up subroutines to make use of the high resolution capabilities of the Commodore 64. These have included Plotsub and Linesub. Here, we look at a routine that will draw circles, given the centre co-ordinates and radius.

It is impossible to produce a precisely drawn circle on a home computer. The accuracy of the approximation we can make depends on the method used and the length and complexity of the final routine. Circle drawing from BASIC usually involves calculations using either the sine and cosine functions or squares and square roots to produce the co-ordinates of points on the circumference of the circle to be drawn. Both of these methods, however, produce difficulties when we try to implement them in machine code, so let's look at an alternative method, which is particularly suited to a machine code solution.

The method we shall use considers the diameter of a circle to be divided into an equal number of parts, each of width W. At each division we can think of a rod that reaches vertically upwards to a point C, on the circumference of the circle. The diagram shows one such rod, N divisions from the left-hand end of the diameter, AB. By joining A and B to C we form two right-angled triangles, ACD and BCD, as shown on the right.



LIZ DIXON

Using Pythagoras's theorem, we can write down the following expressions from this diagram:

$$AC^2 = AD^2 + CD^2$$
$$CB^2 = DB^2 + CD^2$$

If we add these equations together we get:

$$AC^2 + CB^2 = AD^2 + DB^2 + 2CD^2$$

However, it is a special property of circles that triangle ABC is also right-angled. So, we can say:

$$AC^2 + CB^2 = AB^2$$

Putting this into the left-hand side of the earlier equation we get:

$$AB^2 = AD^2 + DB^2 + 2CD^2$$

CD corresponds to 'yup' — the distance from the diameter to the circumference. AD is N×W and AB is 2×R, where R is the radius of the circle. Substituting these values into the equation and rearranging these factors we get:

$$2yup^2 = (2R)^2 - (NW)^2 - (2R-NW)^2$$
$$yup^2 = 2RNW - N^2\,W^2$$

If we decide to divide the diameter into 64 equal parts, then W = 2×R/64, which reduces to W = R/32. Substituting this into the equation:

$$yup^2 = 2RNR/32 - N^2\,R^2/32^2$$
$$yup^2 = R^2/32^2 \times (64N-N^2)$$

and finding the square root of both sides gives:

$$yup = R/32\times SQR(64N - N^2)$$

If we start with x = xcentre − R and increment in 64 equal steps, then at each increment the distance up to the circumference will be given by the formula we have arrived at, where N is the step number. Although this result includes a square root, we should note that the expression to be square-rooted is independent of the centre co-ordinates or radius of the circle. We can, therefore, calculate a table of values for the square root function, which gives a solution for each value of N from 0 to 64. This needs to be done only once and can be incorporated into our program as a 'look-up' table.

The absolute y co-ordinate for each increment of x is:

$$ya = ycentre - yup$$

We can also make use of the symmetry of the circle