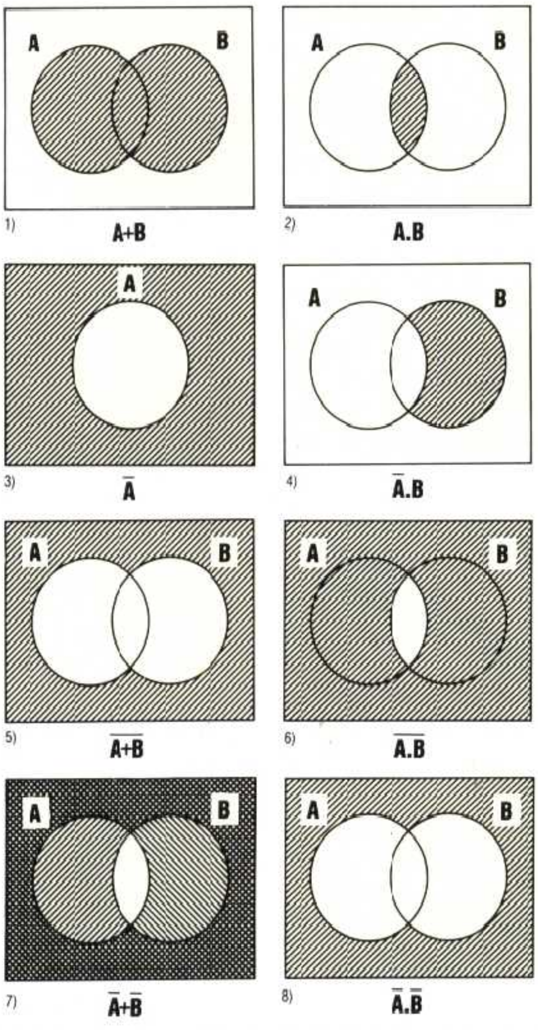
REFINING THE PROCESS

Simplification of Boolean algebra expressions involves the production of equivalent expressions that contain fewer operators (AND, OR or NOT). This simplification is of major importance in logic circuit design as it provides clearly defined methods by which the circuit may be improved in terms of layout and economy.

Venn diagrams provide a useful graphical aid to the simplification of Boolean algebra expressions by allowing simple expressions to be drawn as areas of shading. The area inside a rectangle (symbolised by 1 = Identity or Universal Set) represents all the possible combinations of truth values of the inputs, and circles within the rectangle represent certain combinations. Here we show some represented as Venn diagrams:



1) A+B
2) A.B
3) $\overline{A}$
4) $\overline{A}$.B
5) $\overline{A+B}$
6) $\overline{A.B}$
7) $\overline{A}+\overline{B}$
8) $\overline{A}.\overline{B}$

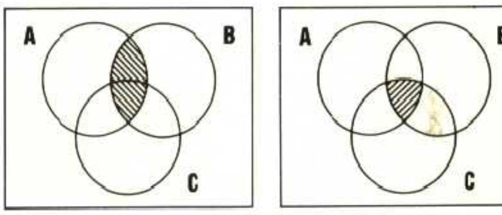
Comparing diagrams 5 and 7 shows at a glance that NOT(A OR B) is not the same as NOT(A) OR NOT(B). Similarly, diagrams 6 and 8 demonstrate that NOT(A AND B) is not equivalent to NOT(A) AND NOT(B).

Perhaps the easiest way to think of AND and OR in terms of Venn diagrams is to think of A.B as the area where area A 'overlaps' with area B; and to regard A+B as the combined areas of A and B. There are several self-evident relations that exist in Boolean algebra. For each of these relations you may try to construct a Venn diagram as proof that they are true. (0 represents the 'empty set' — that is to say an impossible combination.)

1) $A.A = A$
2) $A.\overline{A} = 0$
3) $A.0 = 0$
4) $A.1 = A$
5) $A.(A + B) = A$
6) $A.(\overline{A} + B) = A.B$

LAWS OF BOOLEAN ALGEBRA

The concept of *duality* is an intriguing and useful aid to simplification, relying on the symmetry of the operators AND and OR. To form the dual of any true relation change all the ANDs to ORs and vice versa, and likewise all the noughts to ones and vice versa. For example, let us take the fifth relation mentioned in the preceding list. The dual of this relation is $A + A.B = A$. This expression is also true. It demonstrates another important principle, namely *absorption*. Looking at a Venn diagram, it is clear to see that the A.B term lies wholly within A, and thus can be said to have been absorbed by A. This idea can be extended to a three-variable case, such as $A.B + A.B.C = A.B$. The following pair of Venn diagrams shows that this is true.



A.B   A.B.C

ALISON FENTON

You may like to try writing down the duals of the other five special relations and draw Venn diagrams to confirm their validity.

Look again at the Venn diagrams given earlier. Comparing diagrams 5 and 8 shows us that the following important relation is always true: $\overline{A + B} = \overline{A}.\overline{B}$. Comparing diagrams 6 and 7 shows us that: $\overline{A.B} = \overline{A} + \overline{B}$. These two relationships are known as *de Morgan's Laws* and may be applied in more complex cases such as the case of three