Answers to Exercise 3

1a) $A.(\overline{A}+B)$

$= A.\overline{A} + A.B.$ (distributive law)

$= A.\overline{B}$ ($A.\overline{A}=0$)

b) $X+Y.(X+Y)+X.(\overline{X}+Y)$

$=X+Y+X.(\overline{X}+Y)$ (relation 5)

$=X+Y+X.Y$ (relation 6)

$=X+Y$ (absorption)

c) $P.Q+\overline{P}.Q+\overline{P}.\overline{Q}$

$=P.Q+\overline{P}.(Q+\overline{Q})$ (distributive law)

$=P.Q+\overline{P}$ ($Q+\overline{Q}=1$)

$=\overline{P}+Q$ (dual of relation 6)

d) $\overline{X+Y.Z+Z.Y}$

$=\overline{X}.\overline{Y.Z}.\overline{Z.Y}$ (de Morgan)

$=X.Y.Z.(Z+\overline{Y})$ ($\overline{X}=X$, de Morgan)

$=X.Y.Z.Z+X.Y.Z.\overline{Y}$ (distributive law)

$=X.Y.Z+0$ ($Z.Z=Z$, $Y.\overline{Y}=0$)

$=X.Y.Z$

2) The truth table for the alarm system is:

INPUTS			OUTPUTS
A	B	S	Alarm
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

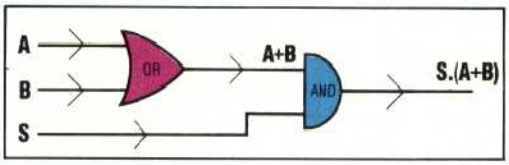
Alarm $= \overline{A}.B.S+A.\overline{B}.S+A.B.S$

$=\overline{A}.B.S+A.S.(\overline{B}+B)$ (distributive law)

$=\overline{A}.B.S+A.S$ ($\overline{B}+B=1$)

$=S.(A+\overline{A}.B)$ (distributive law)

$=S.(A+B)$ (dual of relation 6)



3) If the three switches are X, Y and Z and the hall light is P then the truth table is:

INPUTS			OUTPUTS
X	Y	Z	P
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$P=\overline{X}.\overline{Y}.Z+\overline{X}.Y.\overline{Z}+X.\overline{Y}.\overline{Z}+X.Y.Z$

$=Z.(\overline{X}.\overline{Y}+X.Y)+\overline{Z}.(\overline{X}.Y+X.\overline{Y})$ (distributive law)

$=Z.(\overline{X.Y}+X.\overline{Y})+\overline{Z}.(\overline{X}.Y+X.\overline{Y})$ (de Morgan)

4) The given truth table shows that the question "Do you tell the truth?" is of little use to us because both a liar and a truth-teller will give the same reply. The truth table has the same form as the function $X.Y+\overline{X}.Y$, which simplifies to Y. That is, the answer is dependent on only one variable, not two, so the question does not differentiate between liars and truth-tellers. However, if we ask the question, "Do pigs have wings?" then the table is:

		POSSIBLE ANSWER	
		YES	NO
POSSIBLE IDENTITY OF RESPONDENT	LIAR	1	0
	TRUTH TELLER	0	1

and this is the truth table for the function $X.\overline{Y}+\overline{X}.Y$, which is also an Exclusive-OR table. This question enables us to identify the respondent.