SPATIAL AWARENESS

LOGO is a particularly useful language for investigating pattern and symmetry. We show you how a range of spatial transformations can be performed by the turtle, and develop a procedure that can alter other procedures, and will enable us to create strip patterns.

There are four kinds of transformation that we can apply to a two-dimensional figure and leave its shape unchanged (though its position may change). These transformations are: translation, rotation, reflection and glide reflection. Our diagram shows how a shape's position is changed by each of these transformations.

A figure is said to be *symmetrical* if we can transform it in one or more of these ways and leave its position, as well as its shape, unchanged. Finite shapes (such as polygons and letters of the alphabet) must have symmetries based on reflection and rotation, since translations and glide reflections will change their positions.

To investigate these symmetries it is useful to have LOGO procedures for reflecting and rotating shapes. We'll begin by looking at the task of reflecting a shape in a line that goes through the origin and has a given heading.

It is easiest if we assume that the procedure to draw the shape is state transparent (that is, it leaves the turtle in the same position with the same heading as it had before the procedure was run). Our task then breaks down into two parts: firstly, we need to find the co-ordinates and heading of the starting point of the reflection that corresponds to the starting point of the original shape. The second task needs to be performed before we start drawing the shape. It simply involves changing all right turns in the shape-drawing procedure to left turns, and all left turns to right turns. One way to do this is to replace all RTs and LTs in the procedure with a procedure called TURN, defined as follows:

```
TO TURN :A
  RT :DIR * :A
END
```

So we can now define a square as:

```
REPEAT 4 [FD 50 TURN 90]
```

To use this procedure we must first set the global variable DIR to 1. Thus MAKE "DIR 1 SQUARE will draw a square. To reflect the square in the y-axis all we need to do is type MAKE "DIR (-1) and then SQUARE. Try it and see what happens.

The procedure to position the turtle prior to drawing the reflection depends on a little bit of trigonometry:

```
TO REFLECT :A
  MAKE "H HEADING
  MAKE "XOLD XCOR
  MAKE "ANGLE ( ATAN :YOLD :XOLD ) - 90 + :A
  MAKE "R SQRT ( :XOLD * :XOLD + :YOLD * :YOLD )
  PU
  SETXY 0 0
  SETH :A + :ANGLE
  FD :R
  SETH 2 * :A - :H
  PD
  MAKE "DIR :DIR * (-1)
END
```

This procedure can now be used to see the effect of reflections in various lines through the origin. Try:

```
MAKE "DIR 1
PU SETXY 40 70 PD
SQUARE
REFLECT 60
SQUARE
```

If the reflected shape lies completely on top of the original then it is said to have 'reflective symmetry' about that line. Try:

```
MAKE "DIR 1
PU SETXY 0 0 PD
SQUARE
REFLECT 45
SQUARE
```

A similar procedure could be written to rotate a shape about a given point through a given angle, but we'll leave that for you to write.

Some patterns, such as those on wallpapers, use the same shape repeatedly in their design. It is possible to have translations and glide reflections that move the whole pattern, and yet leave it exactly as it was. For the moment, we'll concentrate on patterns involving translations along a single line, leaving two-dimensional patterns for the next instalment.

Combinations of the fundamental four transformations give rise to just seven kinds of patterns on a straight line. All these possibilities are shown in terms of a simple 'LEG' motif in our second diagram. We have built up procedures for drawing the seven patterns from any MOTIF using the procedures MOVE for translation, TURN for rotation, and R.MOTIF, which changes all the RT turns in MOTIF to LT turns, and all the LT turns to RT.

We have used LOGO's list processing facilities to write the procedure R.MOTIF by rewriting MOTIF. The procedure we use to do this is: