Taking the cases when S = 1, an expression for S can be formed from the truth table:

$$S = \overline{P}.\overline{X}.Y + \overline{P}.X.\overline{Y} + P.\overline{X}.\overline{Y} + P.X.Y$$

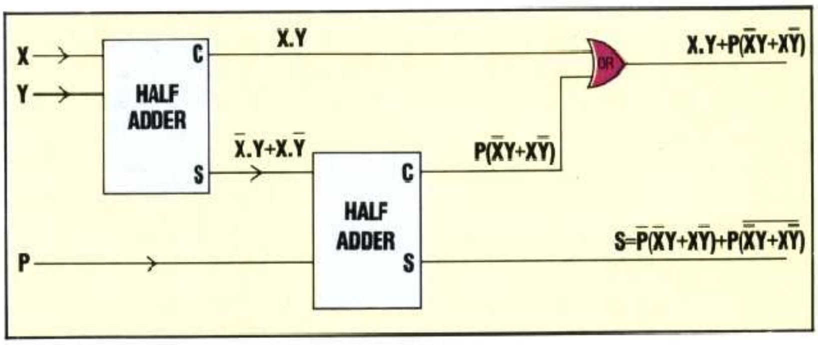
Using the rules we have learned we can simplify this expression:

$$S = \overline{P}.(\ \overline{X}.Y + X.\overline{Y}\ ) + P.(\ \overline{X}.\overline{Y} + X.Y\ )$$
(distributive law)
$$S = \overline{P}.(\ \overline{X}.Y + X.\overline{Y}\ ) + P.(\ \overline{\overline{X}.Y + X.\overline{Y}}\ )$$
(de Morgan)

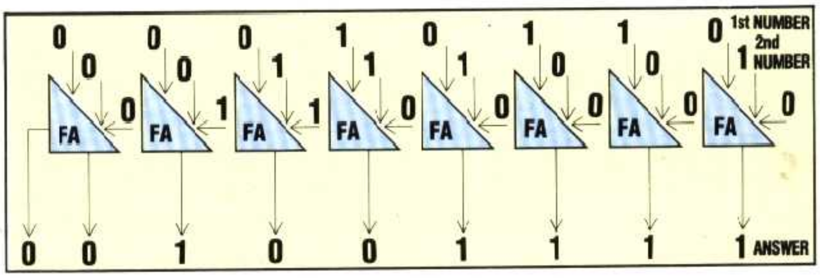
Similarly, we can form an expression for C. From the truth table:

$$C = \overline{P}.X.Y + P.\overline{X}.Y + P.X.\overline{Y} + P.X.Y$$
$$C = X.Y.(\ \overline{P} + P\ ) + P.(\ \overline{X}.Y + X.\overline{Y}\ )$$
(distributive law)
$$C = X.Y + P.(\ \overline{X}.Y + X.\overline{Y}\ )$$
$(\ \overline{P} + P = 1\ )$

Note that $\overline{X}.Y + X.\overline{Y}$ is the sum output from a half adder circuit. Thus, a full adder circuit can be designed from two half adders.



The following example shows how a series of eight full adders combine inside the ALU to perform the binary addition of two eight-bit numbers.



EXERCISE 3

1) Simplify these expressions:

a) $A.(\ \overline{A} + \overline{B}\ )$

b) $X + Y.(\ X + Y\ ) + X.(\ \overline{X} + Y\ )$

c) $P.Q + \overline{P}.Q + \overline{P}.\overline{Q}$

d) $\overline{\overline{X} + \overline{Y}.Z + \overline{Z}.Y}$

2) A car alarm has an on/off switch and switches on the two front doors. The alarm will sound if either or both doors are opened when the on/off switch is set to on. Draw a truth table showing the three inputs (door A, door B, and the on/off switch) and the alarm output. Use your truth table to write a Boolean expression for the alarm sounding and draw a logic circuit for the alarm system.

3) A hall light is operated from a switch at the door, a switch at the bottom of the stairs or one at the top of the stairs. Design a suitable logic circuit.

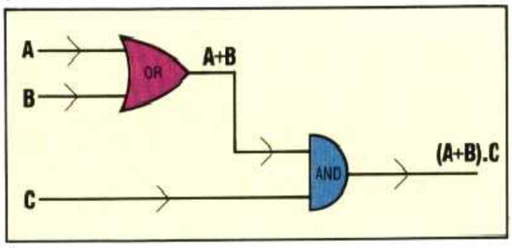
4) You are marooned on a desert island with two other people. One of these people always speaks the truth, the other always tells lies. For your own survival it is imperative that you find out which one will tell the truth. There are a number of questions that you could ask one of them to determine his or her identity. Draw up truth tables to investigate the possible answers. Here is an example to start you off.

'Do you always tell the truth?'

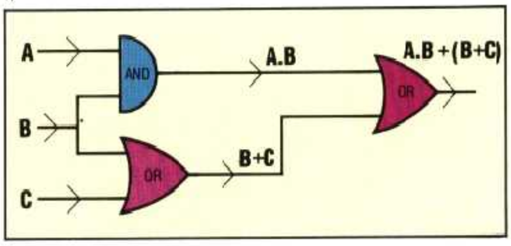
		POSSIBLE ANSWERS	
		YES	NO
POSSIBLE IDENTITY OF RESPONDENT	LIAR	1	0
	TRUTH-TELLER	1	0

Answers to Exercise 2 on page 33

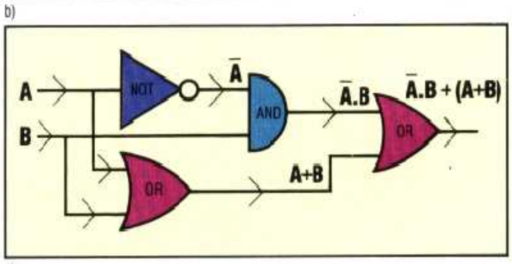
1)



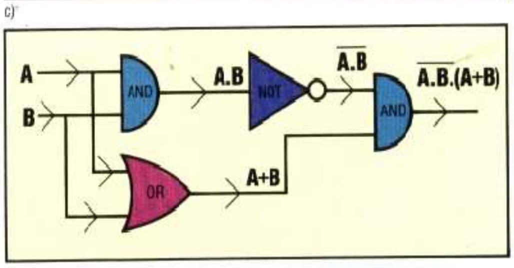
a)



b)



c)



d)

2a) $C = A.B$

b) $S = \overline{A.B}(\ A + B\ )$

3a) $X = (\ A + B\ ).(\ B.C + C\ )$

b) $X = \overline{A.B} + \overline{B}$