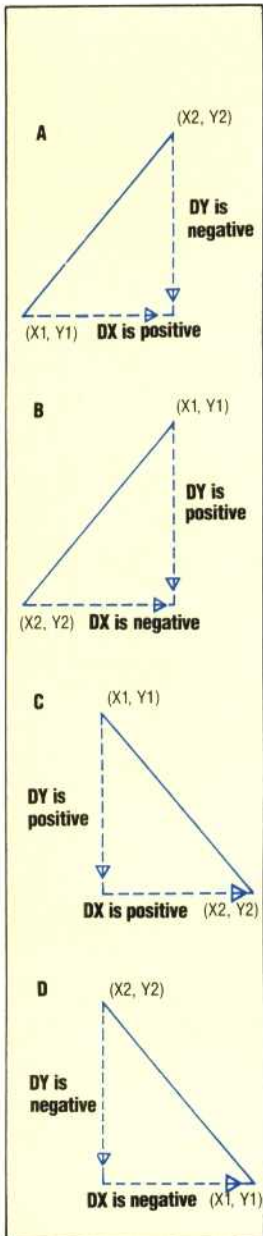




# MAKING THE GRADE



## Gradient One In Four

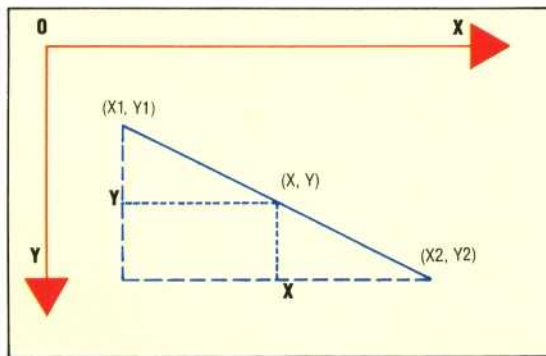
When the absolute value of the gradient,  $(Y_2 - Y_1) / (X_2 - X_1)$ , is greater than one, it is convenient to keep DX positive, by using one of these algorithms:

- Recalculate  $-(DY)$  and decrement Y from  $(X_1, Y_1)$  until  $Y_2$  is reached.
- Swap  $(X_1, Y_1)$  with  $(X_2, Y_2)$  and recalculate.
- Increment Y from  $(X_1, Y_1)$  until  $Y_2$  is reached.
- Swap  $(X_1, Y_1)$  with  $(X_2, Y_2)$  and recalculate.

Earlier in the machine code course we developed a program called Plotsub for the Commodore 64, which allowed us to plot high resolution points (see page 337). Here, we take a look at another routine for the same machine that makes use of Plotsub to draw lines of various gradients.

The most obvious way to draw a line from one point to another on the screen is to calculate the gradient of the line using the co-ordinates of the two endpoints. Thus, starting at one end of the line, we could plot each successive point by incrementing the X co-ordinate, and calculating the corresponding value of the Y co-ordinate using the value of the gradient. Each point of the line would be plotted until the other endpoint was reached. Let's look at the mathematics of this technique in more detail.

If we call the endpoints of the line  $(X_1, Y_1)$  and  $(X_2, Y_2)$ , then the gradient, G, will be equal to  $(Y_2 - Y_1) / (X_2 - X_1)$ . The diagram shows a line with one of the points on it labelled  $(X, Y)$ :



The gradient between the starting point and  $(X, Y)$  is given by calculating  $(Y - Y_1) / (X - X_1)$ . This gradient is, of course, equal to the gradient for the whole line, G. By equating these two elements, and rearranging their terms, we can arrive at a mathematical expression for the Y co-ordinate of any point on the line:

$$\begin{aligned} (Y - Y_1) / (X - X_1) &= G \\ (Y - Y_1) &= G(X - X_1) \\ Y &= Y_1 + G(X - X_1) \end{aligned}$$

If the step between X and  $X_1$  is only one unit (i.e. if we are incrementing the X value by one), then this reduces to:

$$Y = Y_1 + G$$

Therefore, starting at the endpoint  $(X_1, Y_1)$ , and incrementing X, each succeeding value of Y can be calculated by repeatedly adding the value of the

line's gradient.

You may like to try implementing this technique in BASIC using Plotsub to plot each point. However, there are difficulties with this method:

- If the value of  $X_2$  is less than that of  $X_1$ , then we will need to decrement X rather than increment it.
- If the gradient is greater than one, then the line will not be 'continuous'. This is because when G is greater than one, Y will increase by more than one for each increment of X.
- Negative gradients will cause difficulties when implemented in machine code. Two's complement arithmetic could be used to represent negative changes in X and Y, but as the Y co-ordinates can range from 0 to 199 and X values range from 0 to 319, then two byte two's complement arithmetic would be needed.
- Vertical lines cannot be drawn, as calculating their gradients involves division by zero.

## MAKING IMPROVEMENTS

The first problem is easily overcome by swapping the two points before performing any calculations.

To draw a continuous line we need to increment the value of the X co-ordinate and calculate Y if the gradient is less than one, and increment Y and calculate X for gradients greater than one. Splitting the problem according to this condition ( $G > 1$ ?) allows us to avoid having to use division and double byte two's complement arithmetic.

We've already outlined the plotting procedure for those cases where the gradient is less than one; let's consider what must be done when the gradient exceeds one. Let  $DX = X_2 - X_1$  and  $DY = Y_2 - Y_1$ . We can easily determine whether or not the gradient exceeds one by comparing the values of DX and DY. If DX is less than DY, then the gradient is greater than one.

If we wish to avoid negative values of DX and DY, the procedure is more complex. We must consider four possible cases where the gradient is greater than one; these are shown in the diagram. In our program, we must first determine exactly which case we have, and then take the corresponding course of action (assuming that we wish to keep DX positive):

- Recalculate DY as  $Y_1 - Y_2$ . Starting at  $(X_1, Y_1)$ , we decrement Y until  $Y_2$  is reached.
- Swap points and restart.
- Start at  $(X_1, Y_1)$  and increment Y until  $Y_2$  is reached.
- Swap points and restart.

At the beginning of our program we shall calculate DX and DY. Bits within a certain location can be set accordingly if the calculation of DX or DY